Background for the model:
The model tries to determine optimal allocation of natural gas in the time horizon between 1 and 3 years by minimizing both
• shortages of natural gas and
• the direct costs of supplying natural gas.
In order to obtain these goals, the time horizon is divided into smaller decision time periods whose durations depend on the following:
• The uncertainty in future weather conditions
• The demand for natural gas due to changes in weather conditions
• The expected demand for natural gas that can be estimated through “reasonable extrapolation of past data” by taking consideration of
  1. availability of natural gas
  2. population growth
  3. per capita income

The construction of the model:
A single natural gas utility supplies natural gas to a network of interconnected geographical regions. The size of each geographical region depends on
• the amount and varieties of energy required by that region and
• the service area of natural gas utility.
The natural gas utility buys natural gas from three different sources:
• Regular sources such as transmission line companies and local producers
• Nonregular sources for emergency purchases
• Gasifying coal plants that produce synthetic natural gas (not available at this time)
The natural gas utility sells natural gas to three different consumers:
• Residential consumers such as homes
• Commercial consumers such as businesses
• Industrial consumers such as manufacturers
Consider the figure below:
The figure explains how much natural gas flows from the suppliers to the consumers in a service area supplied by a single natural gas utility during a single time period.

**Linear Problem P(r) of the model:**
Find the optimal purchase of natural gas by the natural gas utility with respect to the demand forecasted during the end of time period \( r \) for demands during following periods.

**Variables of the model:**
Let MMCF be one million standard cubic feet at \( 60^0 \) F and \( i = \{ a, b, c \} \). Then,

\[
X(g, t) = \text{quantity of natural gas (MMCF/week) to be allocated from storage facility of utility } g \text{ to consumers during time period } t.
\]

\[
Y(i; g; t, t+u(i,t)) = \text{quantity of natural gas (MMCF/week) ordered by utility } g \text{ from source } i, \text{ during time period } t, \text{ and to be obtained and allocated to consumers during time period } t + u(i, t).
\]

\[
Z(i; g; t, t+u(i,t)) = \text{quantities of natural gas (MMCF) ordered by utility } g, \text{ from source } i, \text{ during time period } t, \text{ to be obtained and stored during time period } t + u(i, t).
\]

\[
T(t) = \text{length of time period } t \text{ in weeks}
\]

\[
I(t) = \text{total working gas (MMCF) in storage facility of utility } g, \text{ at the end of period } t, \text{ after satisfying the requirements in period } t.
\]

\[
I(r) = \text{the inventory at the end of time period } r.
\]

\[
D(r; k; t) = \text{the total demand for natural gas (MMCF/week) during time period } t, \text{ by consumers in geographical region } k \text{ which was forecasted at the end of time period } r (r < t).
\]
Objective functions of the model:
1. Minimizing the expected total shortage of natural gas in a service area supplied by utility g, for all periods following period r.
   \[ \min SH(g; r) = \sum T(t) [\sum D(r,k,t) - \sum (\sum Y(i; g; j, t) + \sum Y(i; g; j, j+u(i,t)))] - X(g, t) \]
2. Minimizing expected cost of natural gas (direct cost of buying it and storing it) to utility g, for all periods following period r.
   \[ \min W(g; r) = \sum [T(t) \sum C(i; g; t) \left( Y(i; g; t, t+u(i,t)) + Z(i; g; t, t+u(i,t)) + f(g; t) I(t) \right)] \]
   where
   \[ C(i; g; t) = \text{unit cost ($/MMCF$) of buying natural gas from source i by utility g during time period t.} \]
   \[ f(g; t) = \text{cost to store one MMCF of natural gas from time period t to time period t+1 in storage facility belonging to utility g.} \]

Constraints of the model:
- **Demand Constraints** for consumers in geographical region k, within the service area of single utility g, during time period t (t > r) are given by
  \[ a(g; k; t) \left( \sum \left[ \sum Y(i; g; j, t) + \sum Y(i; g; j, j+u(i,j)) \right] + X(g, t) \right) \leq D(r; k; t) \text{ for } 1 \leq j \leq r \]
- **Material Balance Equations** for relating storage variables to the ordering variables.
  1. \[ I(t)=I(t-1) + T(t) \left[ \sum Z(i; g; j, t) + \sum Z(i; g; j, j+u(i,j)) \right] - X(g, t) \text{ for } t > r \]
  2. \[ I(r)=I(r-1) + T(t) \left[ \sum Z(i; g; j, r) \right] - X(g, r) \]
     where
     \[ I(0)=\text{initial inventory,} \]
     \[ I(M)=\text{desired ending inventory.} \]
- **Storage Constraints**
  1. Let V(g,t) be the total working gas capacity (MMCF) of utility g at the beginning of time period t. Then, storage capacity constraint is
     \[ I(t-1) + e \left[ \sum Z(i; g; j, t) + \sum Z(i; g; j, j+u(i,j)) \right] - X(g, t) \leq V(g,t) \text{ for } e=1,2,\ldots,T(t) \text{ and } t > r \]
  2. The associated flow constraint is
     \[ X(g, t) \leq I(t-1) + (e - 1) \left[ \sum Z(i; g; j, t) + \sum Z(i; g; j, j+u(i,j)) \right] - X(g, t) \text{ for } e=1,2,\ldots,T(t) \text{ and } t > r \]
  3. Let FI(g) be the maximum possible rate of flow (MMCF/day) **into** storage facility of utility g. Then,
     \[ \frac{1}{7} \sum [\sum Z(i; g; j, j) \sum Z(i; g; j, j+u(i,j))] \leq \text{FI(g)} \text{ for } t > r \]
  4. Let FO(g) be the maximum possible rate of flow (MMCF/day) **out of** storage facility of utility g. Then,
     \[ \frac{1}{7} X(g,t) \leq \text{FO(g)} I(t-1) \]
• **Supply Constraints**
  Let $S(i; g; t)$ be the quantity of natural gas (MMCF/week) available at source $i$ during time period $t$ for supply to utility $g$. Then,
  \[
  \sum [Y(i; g; j, t+u(i,t))+Z(i; g; j, t+u(i,t))] + Y(i; g; t, t+u(i,t))+Z(i; g; t, t+u(i,t)) \leq S(i; g; t)
  \]

• **Interstate Pipeline Rate of Flow Constraints**
  Let $PF(b)$ be the maximum possible rate of flow (MMCF/day) in the interstate pipeline connecting nonregular source $b$ and utility $g$. Then,
  \[
  \left(\frac{1}{7}\right) \sum \left[ \sum [Y(b; g; j, t)+Z(b; g; j, t)] + \sum [Y(b; g; j, j+u(i,j))+Z(b; g; j, j+u(i,j))] \right] \leq PF(b)
  \]
  for all $b$ and $t > r$ and where $G$ is the set of all natural gas utilities using the same interstate pipeline connecting $b$ to utility in question.

• **Nonnegativity Constraints** All variables are nonnegative.

**Solution to the LP problem in the model:**
In the model, LP has two opposing and unrelated objective functions. To solve LP, we assign a parametric value to the total shortage of natural gas objective function, turning it into a parametric constraint. Thus, the optimal solution will be the least cost for the least shortage of natural gas.

This model was tested by East Ohio Gas Company. The following results were found:
• Minimum average direct cost of natural gas decreases with
  1. increasing total annual shortage of natural gas
  2. increasing total storage capacity until a certain point
• The curves that show the minimum average direct cost of natural gas as a function of the total annual shortage are approximately linear.
• The curves for different storage capacities are parallel to each other.